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Azimuthal pion fluctuation and phase transition in ultra-relativistic ring-like and jet-like events

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bstract : This paper presents an analysis of ring-like events and jet-like events in ^{16}O-AgBr interactions at 60 A GeV in terms of Scaled Factorial oments (SFMs) in azimuthal space upto the sixth order of moments The analysis reveals that non-thermal phase transition is clearly signalled by in case of jet like events No such phase transition is observed in ring-like events

eywo : Non thermal phase transition, ultra-relativistic nucleus-nucleus collisions, intermittency, ring-like events and jet-like events

25 75 q 25 70 Mn, 24 60 Ky

1 Introduction

Fluctuation studies in the distribution of produced particles ncapsulate rich information about the dynamics of the emitting ource in the late stage of a nucleus-nucleus interaction where he nuclear matter is highly excited and diffused Unusually, arge density fluctuations in (pseudo) rapidity have been served in cosmic events [1] as well as in hadron-hadron [2,3], dron nucleus and nucleus-nucleus [4] collisions Last few ars have witnessed a remarkably intense experimental and eoretical activity in search of scale invariant fluctuations in ultihadron production processes, commonly known as ntermittency' 'Intermittency' refers to the power-law type of haviour of normalized scaled factorial moments with creasing phase space This phenomenon is designed for parating non-statistical fluctuations from the statistical part ialas and Peschanski [5] first proposed this formalism in a neering work in course of their analysis of the distribution produced particles in cosmic ray events [1]. Subsequent alyses [6-12] of high-energy data of various collision cesses in the available accelerator energy range show that intermittency appears to be a general phenomenon in multiparticle production as well as target fragmentation process in high energy interactions When the behavior of the scaled factorial moments appeared to be inconsistent with the standard models of particle production, the concept of self-similarity in the random cascade, or α -model [5, 13] was introduced to explain this phenomenon The possible existence of self-similarity led to further studies [14, 15] in terms of generalized dimensions of fractal geometry [16] already discussed in other branches of physics Fractals are self-similar objects of a non-integral dimension If a single exponent can describe these properties of self-similarity, one has a simple or homogeneous fractal In a more complex case, the term multi-fractality is used when the scaling properties may be different for different regions of the system The intermittency exponent α_q can be related to the anomalous fractal dimension d_q as $d_q = \alpha_q/(q-1)$ [17], where q is the order of the moment

The properties of intermittency have been studied [18] to investigate the structure of different phases of self-similar multi-particle system using the random cascade model According to Peschanski [19], a self-similar cascading might lead to a possible occurrence of a non-thermal phase transition for multi-particle production The signals of non-thermal phase transition can be studied with the help of the function $\lambda_q = (\alpha_q + 1)/q$, where

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α_q is the q-th order of intermittency exponent [18] The condition for the existence of different phases of a self-similar cascade is that λ_q has a minimum value at $q = q_c$ [18-20], where q_c is the critical value of q It has been conjectured that in the regions $q < q_c$ and $q > q_c$, self-similar multiparticle systems behave differently [20] The region $q < q_c$ is dominated by numerous fluctuations where one bin contains particles not more than q_c, whereas for the region $q > q_c$, it is governed sometimes by a small number of very large fluctuations This situation resembles a mixture of a 'liquid' of many small fluctuations and a 'dust' consisting of a few grains of very high density The 'liquid' and the 'dust' coexist When the system is probed by a moment of rank $q < q_c$, one sees only the liquid phase When the system is probed by a moment of $q > q_c$, one sees only the dust phase

From analysis of azimuthal distribution of pions in ultra-relativistic nuclear collisions, two different classes of substructures are found which are referred as jet-like and ring-like structures [21] The 'ring-like structures' are occurring where many pions are produced in narrow regions along the rapidity axis which are at the same time diluted over the whole azimuth The 'jet-like structure' consists of cases where particles are focused in both dimensions [22] The ring-like events have been reported in cosmic rays about 20 years ago and subsequently, in accelerator energy in nucleus-nucleus collisions [23] Although primarily it is observed that jet-like and ring-like events do not exhibit significant deviations from what can be expected from stochastic nature But it is important to realize that the studies of average values do not give full information of the process involved Following the works of Adamovich *et al*[21], we have obtained ring-like and jet-like events in ^{16}O-AgBr interaction at 60 A GeV

In this paper, we have investigated the non-statistical fluctuations in terms of scaled factorial moments in the azimuthal angle distribution of pions in ^{16}O-AgBr interaction at 60 A GeV for both ring-like and jet-like events separately. We use power law behaviour of the factorial moments with respect to decreasing phase space, which in turn, is closely related to the intermittent parameter α_q We search for possibility of occurrence of non-thermal phase transition for both the events in terms of function λ_q

The data used in the present investigation have been obtained from sets of photo emulsion plates exposed to ^{16}O beam with energy 60 A GeV at the CERN SPS Details about the data sets and measuring procedure may be found in Ref [24]

2. Methodology for separation of ring-like and jet-like events

Each consecutive n_d-tuple of particles along the η -axis can then be considered as a group characterized by $\Delta\eta_i$ and $\rho_i = n_d / \Delta\eta_i$ Dense group can then be defined and recorded as above This method has the advantage that all groups, including the discarded, more dilute ones, have by definition the same multiplicity n_d, and can be readily compared With this method, it is also a fairly simple task to compare the obtained sample with samples obtained by a purely stochastic process as well as samples obtained from model-based Monte Carlo calculations

Next, we need to parameterize the azimuthal structure in a suitable way, so that large values of the parameter represent one type of structure and small values the other Two sums have been suggested as such parameters and are given by

$$S_1 = -\sum \ln(\Delta\phi_i)$$

and

$$S_2 = \sum(\Delta\phi_i)^2,$$

where $\Delta\phi_i$ is the azimuthal difference between neighbouring particles in the group For the sake of simplicity we can count $\Delta\phi_i$ in units of full revolutions and thus we have

$$\sum(\Delta\phi_i) = 1$$

Both theses parameters will be large $(S_1 \to \infty, S_2 \to 1)$ for jet-like structures and small $(S_1 \to n_d, S_2 \to 1/n_d)$ for ring-like structures S_2 distribution is used to separate ring-like and jet-like events

3. Scaled factorial moment (SFM) method

Intermittency is defined in terms of scaled factorial moments in azimuthal angle space $(\Delta\phi)$, for each event, we calculate normalised scaled factorial moments as [13]

$$F_q(\delta\phi) = M^{q-1} \sum_{m=1}^{M} \frac{n_m(n_m - 1) \dots (n_m - q + 1)}{\langle n_m \rangle^q},$$

where the full widths of azimuthal angle space $(\Delta\phi)$ is divided into M bins, each of size $\delta\phi = \Delta\phi / M$, n_m is the multiplicity in the m^{th} bin, m running from 1 to M, q is a positive integer which indicates the order of moment For given q and M values, F_q are calculated for all the events and then averaged over the whole sample of events to obtain $\langle F_q \rangle$ The unique feature of this moment is that it can detect and characterize the non-statistical density fluctuations in particle spectra, which is intimately connected with the dynamics of particle production

We have performed our study in one-dimensional (ϕ) space where ϕ is the azimuthal angle with respect to the beam direction The azimuthal angle region used is 0 to 2π As shape of the distribution influences the scaling behaviour of the factorial moments, we have used the 'cumulative' variable [25] X_ϕ instead of ϕ. The corresponding region of investigation for the variable then become (0, 1). The cumulative variable $X(x)$ is given by the relation as below

$$X(x) = \int_{x_1}^{x} \rho(x')\,\partial x' \Big/ \int_{x_1}^{x_2} \rho(x')\,\partial x', \tag{2}$$

where x_1 and x_2 are two extreme points in the distribution $\rho(x)$, between which X varies from 0 to 1. Due to the scale properties of the variables, one particle spectrum stretches in its central region eliminating the losses from the beam splitting and thus allowing to observe the higher order of moments. In our analysis, we will use the above mentioned method.

If the non-statistical fluctuations are self-similar in nature, in the limit of small bin size, factorial moment follow power law behaviour like

$$\langle F_q \rangle \propto M^{\alpha_q}$$

$$\ln\langle F_q \rangle = \alpha_q \ln M + c \tag{3}$$

This property, in analogy with turbulent fluid dynamics, is called 'intermittency'. α_q measures the strength of the intermittency and is called the intermittency exponent and c is a constant. The intermittency exponent α_q is obtained by performing best fits according to eq. (3).

Now non-thermal phase transition λ_q is calculated using the relation

$$\lambda_q = (\alpha_q + 1)/q, \tag{4}$$

where α_q is the intermittency exponent.

Results and discussion

We have calculated SFMs of orders 2, 3, 4, 5 and 6 for the bin range from 6 to 60 in azimuthal angle space using eq. (1). We transformed the variables to reduce the distortion of intermittency due to non-uniformity of single particle distribution using eq. (2) and performed our analysis with the transformed variables. Total ϕ space is considered for the analysis. We have studied the variation of $\langle F_q \rangle$ against M in a log-log plot of different orders (2-6) for ring-like events and jet-like events. To find the best linear behaviour for each interaction, linear fit is performed and χ^2 per degrees of freedom are noted. The intermittency exponents α_q are evaluated by performing the best linear fits according to eq. (3). The values of the intermittency exponent for different orders of moments for shower multiplicity are shown in Tables 1(a,b) for the two groups of events respectively.

Table 1(a) Values of intermittency exponents α_q and λ_q for different orders of moments (q) in azimuthal (ϕ) space for ring-like events

	$q = 2$	$q = 3$	$q = 4$	$q = 5$	$q = 6$
α_q	0.141 ± 0.004	0.437 ± 0.015	0.830 ± 0.038	1.226 ± 0.069	1.587 ± 0.102
λ_q	0.570 ± 0.001	0.479 ± 0.005	0.457 ± 0.009	0.445 ± 0.014	0.431 ± 0.017

Table 1(b). Values of intermittency exponents α_q and λ_q for different orders of moments(q) in azimuthal (ϕ) space for jet-like events

	$q = 2$	$q = 3$	$q = 4$	$q = 5$	$q = 6$
α_q	0.180 ± 0.005	0.568 ± 0.022	1.106 ± 0.050	1.674 ± 0.085	2.216 ± 0.120
λ_q	0.590 ± 0.002	0.522 ± 0.011	0.526 ± 0.025	0.534 ± 0.042	0.536 ± 0.060

Figures 1(a,b) represent the variation of $\ln\langle F_q \rangle$ with $\ln(M)$ for the ring-like and jet-like events, respectively. From Tables 1(a) and 1(b), it is observed that intermittency exponents for jet-like events are significantly larger than those in case of ring-like events indicating stronger fluctuation in azimuthal space.

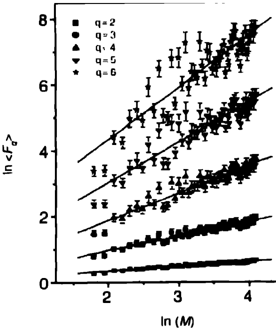


Figure 1(a) Plot of the dependence of logarithm of factorial moments of orders q = 2 to 6 in azimuthal space for ring-like events

Further to study the non-thermal phase transition, we have calculated λ_q from the values of intermittency exponents α_q according to eq. (4) for the two groups of events (Tables 1(a) and 1(b)). The variation of λ_q with order q has been studied. From the plots (Figure 2), it is seen that λ_q shows a minimum at

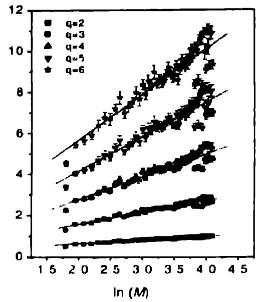


Figure 1(b). Plot of the dependence of logarithm of factorial moments of orders q=2 to 6 in azimuthal space for jet like events

q = 3 for jet-like events But no such minimum is obtained for the ring-like events

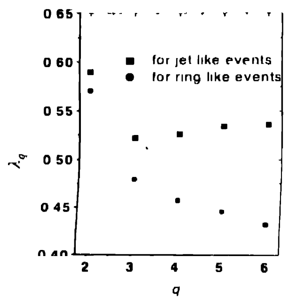
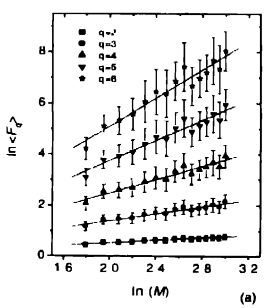


Figure 2 Plot of λ_q for different orders q in azimuthal (ϕ) space for ring-like and jet-like events

In order to get a clear picture of the phase structure of the multiparticle system, we have studied the non-thermal phase transition in detail by sub-dividing the total bin range into a number of sub-bins for both jet and ring-like events



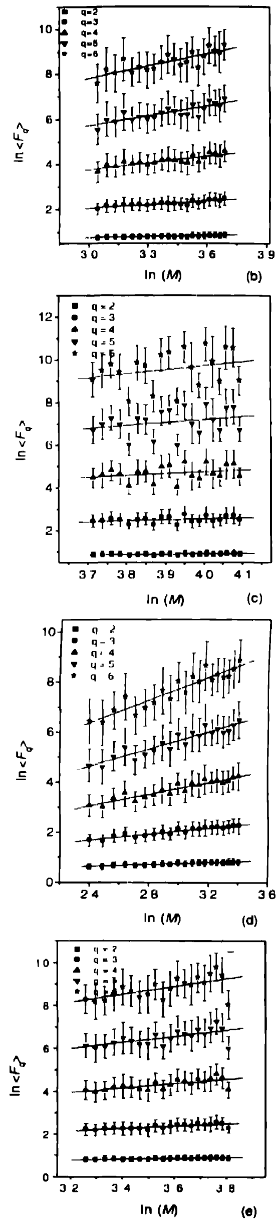


Figure 3. Plots of the dependence of logarithm of factorial momentof orders q = 2 to 6 in azimuthal space (ϕ) for jet- like events for the bin ranges (a) 6 ≤ M ≤ 20, (b) 21≤ M ≤ 40, (c) 41 ≤ M ≤ 60, (d) 11≤ M ≤ 30 and (e) 26 ≤ M ≤ 45 respectively

For jet-like events, we divided the total bin-range into five erent bin-ranges ($6 \le M \le 20$, $21 \le M \le 40$, $41 \le M \le 60$, $11 \le M$ 0 and $26 \le M \le 45$) We have studied the variation of $<F_q>$ inst M in a log-log plot of different orders (q = 2-6) for jet-like nts (Figures 3(a-e)).

From the linear best fits, intermittency exponents are culated (Table 2 (a)) We find from Table 2(b) that χ^2 per grees of freedom values are better than the value responding to the full range λ_q is calculated (Table 2(c)) m α_q and plotted for different orders (q = 2 – 6) for jet-like nts

le 2(a) Values of intermittency exponents α_q for different orders moments (q) for different bin regions in azimuthal (ϕ) space for jet-events

range	q = 2	q = 3	q = 4	q = 5	q = 6
M 20	0 237	0 682	1 289	1 987	2 728
	±	±	±	±	±
	0 015	0 049	0 095	0 144	0 197
M 40	0 157	0 528	1 008	1 457	1 836
	±	±	±	±	±
	0 019	0 066	0 137	0 219	0 301
M 60	0 160	0 440	0 869	1 384	1 899
	±	±	±	±	±
	0 052	0 271	0 675	1 141	1 620
M 30	0 188	0 595	1 154	1 736	2 273
	±	±	±	±	±
	0 012	0 043	0 090	0 145	0 204
M 45	0 170	0 556	1 028	1 448	1 806
	±	±	±	±	±
	0 025	0 100	0 230	0 381	0 533

le 2(b) Variation of χ^2 / degrees of freedom for different orders q different bin regions in azimuthal (ϕ) space jet-like events

range	q = 2	q = 3	q = 4	q = 5	q = 6
$M \le 20$	0 041	0 300	0 118	0 130	0 562
M 40	0 020	0 042	0 454	0 081	0 101
$M < 60$	0 044	0 228	0 578	0 094	1 436
M 30	0 021	0 050	0 069	0 839	0 105
$M \le 45$	0 023	0 076	0 161	2 454	0 314

As mentioned above, the existence of a minimum of λ_q at a rtain critical value of q will assure the transition It is observed m the plots (Figure 4), that minimum λ_q occurs (considering e statistical error at each point) for the intervals $6 \le M \le 20$, $\le M \le 60$ and $11 \le M \le 30$ If we consider the errors in the lues of λ_q, still there is an indication of a dip and then the λ_q values start rising and continue up for the higher value of q As is evident from Figure 4, the values are well-separated from each other for different bins at the higher order This occurrence of minimum λ_q may be attributed to the manifestation of a non-thermal phase transition for the jet-like events in the pionisation region of ^{16}O-AgBr interactions at 60 AGeV

Table 2(c) Variation of λ_q for different orders q for different bin regions in azimuthal(ϕ) space for jet-like events

Bin range	q = 2	q = 3	q = 4	q = 5	q = 6
$6 \le M \le 20$	0 618	0 560	0 572	0 597	0 621
	±	±	±	±	±
	0 007	0 016	0 024	0 029	0 033
$21 \le M \le 40$	0 578	0 509	0 502	0 491	0 471
	±	±	±	±	±
	0 009	0 022	0 034	0 044	0 050
$41 \le M \le 60$	0 580	0 480	0 467	0 476	0 483
	±	±	±	±	±
	0 026	0 090	0 169	0 228	0 270
$11 \le M \le 30$	0 594	0 531	0 538	0 547	0 545
	±	±	±	±	±
	0 006	0 014	0 070	0 093	0 036
$26 \le M < 45$	0 585	0 518	0 507	0 489	0 467
	±	±	±	±	±
	0 012	0 030	0 057	0 076	0 088

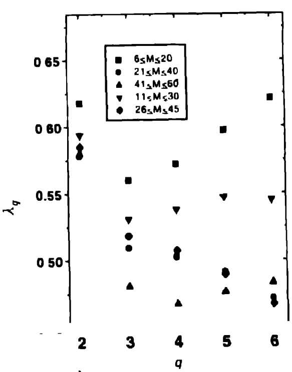


Figure 4. Plot of λ_q for different orders q for different bin regions in azimuthal (ϕ) space for jet-like events

Similarly for ring-like events, we have studied the variation of $<F_q>$ against M in a log-log plot (Figures 5(a-e)) for q = 2–6 in the same sub-bin ranges *i e*. $6 \le M \le 20$, $21 \le M \le 40$, $41 \le M \le 60$, $11 \le M \le 30$ and $26 \le M \le 45$ The slope parameters are

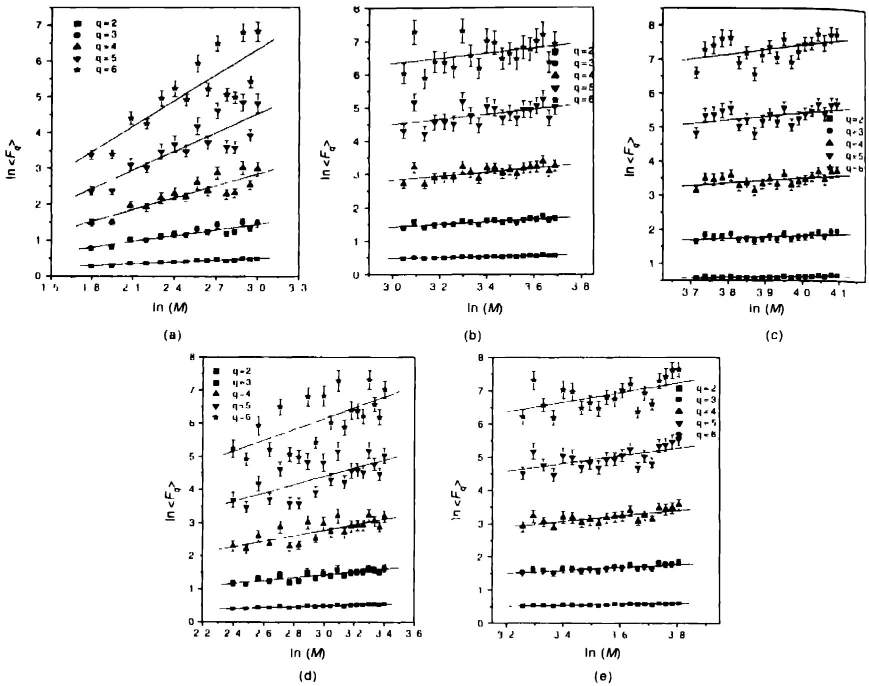


Figure 5. Plots of the dependence of logarithm of factorial moment of orders q = 2 to 6 in azimuthal space (ϕ) for ring- like events for the bin ranges (a) $6 \le M < 20$, (b) $21 \le M < 40$, (c) $41 < M \le 60$, (d) $11 \le M < 30$ and (e) $26 < M < 45$, respectively

calculated from the linear best fits and χ^2 per degrees of freedom are evaluated (Tables 3(a) and 3(b))

Table 3(a) Values of intermittency exponents α_q for different orders of moments (q) for different bin regions in azimuthal (ϕ) space for ring-like events

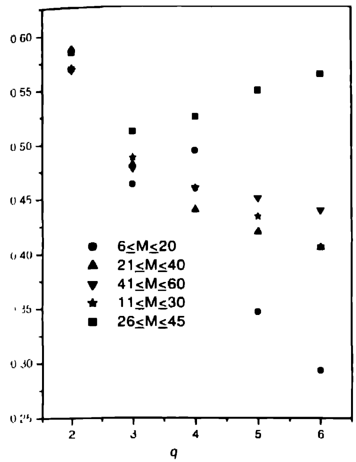
Bin range	q = 2	q = 3	q = 4	q = 5	q = 6
$6 \le M \le 20$	0 168	0 533	1 096	1 742	2 381
	±	±	±	±	±
	0 014	0 061	0 157	0 284	0 421
$21 \le M \le 40$	0 136	0 388	0 614	0 733	0 757
	±	±	±	±	±
	0 014	0 061	0 162	0 300	0 457
$41 \le M \le 60$	0 174	0 441	0 757	1 099	1 433
	±	±	±	±	±
	0 042	0 141	0 285	0 451	0 634
$1 \le M \le 30$	0 137	0 433	0 839	1 254	1 635
	±	±	±	±	±
	0 011	0 059	0 161	0 296	0 442
$26 \le M \le 45$	0 144	0 462	0 837	1 165	1 439
	±	±	±	±	±
	0 016	0 071	0 177	0 327	0 506

Table 3(b). Variation of χ^2 /degrees of freedom for different orders for different bin regions in azimuthal (ϕ) space ring like events

Bin range	q =		q = 4	q = 5	
$6 \le M \le 20$	0 478	1 281	2 697	4 770	7 501
$21 \le M \le 40$	0 122	0 313	0 735	1 513	2 697
$41 \le M \le 60$	0 362	0 623	1 046	1 719	2 707
$11 \le M \le 30$	0 204	0 804	1 948	3 726	6 204
$26 \le M \le 45$	0 121	1 036	0 788	1 665	3 020

Figure 6 reveals the variation of λ_q with the order for different M intervals As it appears from the figure, the minimum of λ_q occurs at $q = 3$ only for the interval $6 \le M \le 20$ However the values of λ_q decreases with the order of moment q in other bins Even considering the errors in λ_q, there is no depression at any point in the plots of λ_q *vs* q for the bins $21 \le M \le 40$ $41 \le M \le 60$, $11 \le M \le 30$ and $26 \le M \le 45$

Thus, one can conclude that in case of jet-like events, the signature of non-thermal phase transition is more clearly pronounced than in case of ring-like events



6 Plot of λ_q for different orders q for different bin regions in ...al (ϕ) space for ring – like events

...(c) Variation of λ_q for different orders q for different bin regions in azimuthal(ϕ) space for ring -like events

...ge	$q = 2$	$q = 3$	$q = 4$	$q = 5$	$q = 6$
20	0 584	0 511	0 524	0 548	0 563
	±	±	±	±	±
	0 007	0 020	0 039	0 057	0 070
– 40	0 568	0 462	0 493	0 346	0 292
	±	±	±	±	±
	0 007	0 020	0 040	0 060	0 076
– 60	0 587	0 480	0 439	0 419	0 405
	±	±	±	±	±
	0 021	0 047	0 071	0 090	0 105
– 30	0 568	0 477	0 459	0 450	0 439
	±	±	±	±	±
	0 005	0 019	0 040	0 059	0 073
45	0 570	0 487	0 459	0 433	0 406
	±	±	±	±	±
	0 008	0 023	0 044	0 065	0 084

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